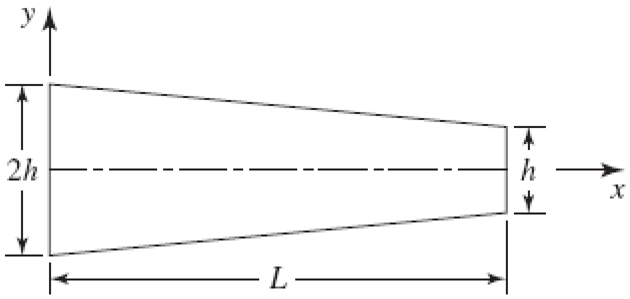
1. The tapered beam element shown in the figure has uniform thickness t and varies linearly in height from $2h$ to h.

   a) Derive strain energy expression for the element.

   b) Derive the value of component k_{11} of the element stiffness matrix by first theorem of Castigliano.



Ref: Fundamentals of finite element analysis, D.V. Hutton

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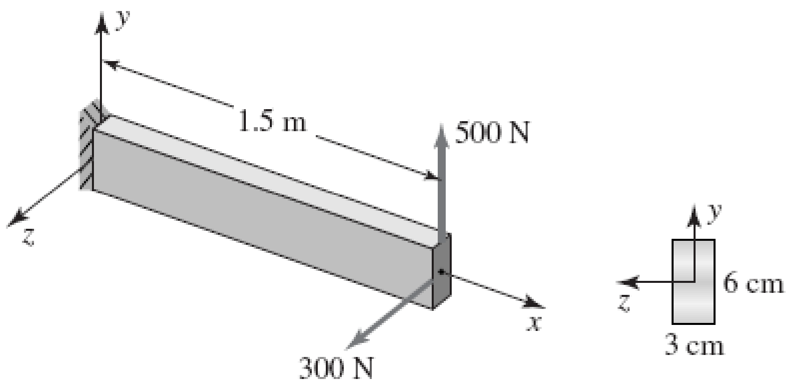
2. The cantilevered beam depicted in the figure is subjected to two-plane bending. The loads are applied such that the planes of bending correspond to the principal moments of inertia. Noting that no axial or tensional loadings are present, model the beam as a single element (that is, construct the 8 × 8 stiffness matrix containing bending terms only) and compute the deflections of the free end, node 2. Determine the exact location and magnitude of the maximum bending stress. (Use E = 207 GPa.)



Ref: Fundamentals of finite element analysis, D.V. Hutton

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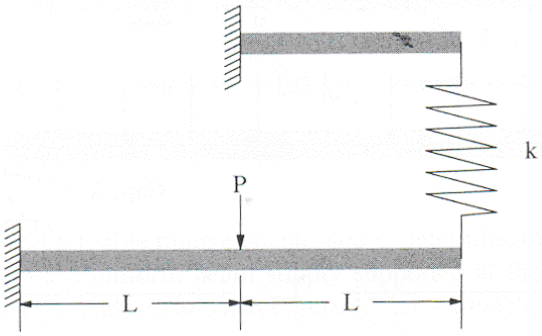
3. The uniform beams are connected together through a spring as shown in the figure. Determine deflections, bending moment, and shear forces in the beams. What is the force in the spring? The numerical values are

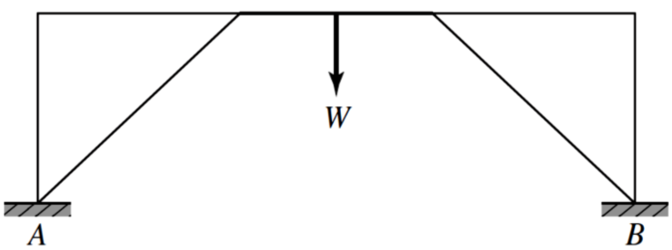
$$E = 10^4 ksi \quad I = 1000 in^4 \quad L = 100 in \quad P = 100 kips \quad k = 2000 kips/in$$

Use kips.in units in calculations.



4. The frame structure shown in the figure is the support structure for a hoist located at the point of application of load *W*. The supports at *A* and *B* are completely fixed. Other connections are welded. Assuming the structure to be modeled using the minimum number of beam-axial elements:

a. How many elements are needed?
b. What is the size of the assembled global stiffness matrix?
c. What are the constraint (boundary) conditions?
d. What is the size of the reduced global stiffness matrix after application of the constraint conditions?



Ref: Fundamentals of finite element analysis, D.V. Hutton